## Pion and photon couplings of $N^*$ resonances from scattering on the proton

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## Abstract

Results of a relativistic model for pion- and photon-induced reactions on the proton are presented. The model is crossing symmetric and gauge invariant. The nucleon resonances  $P_{11}(1440)$ ,  $P_{11}(1710)$ ,  $D_{13}(1520)$ ,  $S_{11}(1535)$ ,  $S_{11}(1650)$ ,  $P_{33}(1232)$ ,  $P_{33}(1600)$ ,  $S_{31}(1620)$ , and  $D_{33}(1700)$  have been included explicitly in the calculation. Unitarity within the channel space  $\pi N \oplus \gamma N$  below the two-pion production threshold has been achieved by using the K-matrix approach.

Strong and electromagnetic coupling parameters of the  $N^*$  resonances have been determined from a fit to the pion-nucleon phase shifts, pion-photoproduction multipoles and Compton-scattering cross sections. The model is shown to describe simultaneously most of the available data. Results for the electric and magnetic polarizabilities of the nucleon are presented.

In recent years effective Lagrangian models have been applied to pion-nucleon scattering  $\pi N \to \pi N$  [1–5], pion photoproduction  $\gamma N \to \pi N$  [6–11], and Compton scattering  $\gamma N \to \gamma N$  [12] on the nucleon. In these calculations where nucleon resonances are explicitly included the interference between the resonant and background contributions plays an important role. From pion photoproduction calculations [13,14,6,11] it is known that the unitarity constraint is important to obtain the correct interference. In most calculations unitarity was imposed by using Watson's theorem in the  $\Delta(1232)$  resonance channel. In Refs. [1,2,4,5]  $\pi N$  rescattering effects were studied in the framework of three-dimensional equations, which also guarantee (two-body) unitarity.

In the case of Compton scattering strong constraints on the amplitude follow from unitarity and causality. These constraints are usually formulated in terms of fixed-t dispersion relations [15–18] by relating the imaginary part of the amplitude via the optical theorem to the pion-photoproduction cross section. Although such a dispersion relation approach has many advantages, unfortunately the link with the decay properties of the nucleon resonances is rather obscure. In particular, it is difficult to extract parameters of the electromagnetic decays of resonances  $N^* \to N\gamma$ .

It is the purpose of this Letter to study all three above processes in an unified approach based on an effective Lagrangian model and unitarity, and to extract the pion and photon couplings of the  $N^*$  resonances below  $\sqrt{s} \simeq 1.7$  GeV. The advantage is that the different processes are calculated consistently, therefore putting stronger constraints on the parameters. In an earlier paper [19] a similar approach was used in the  $\Delta(1232)$  resonance region. Here, energies up to  $\sqrt{s} \simeq 1.7$  GeV are studied and correspondingly the heavier nucleon resonances are also explicitly included. The model is fully relativistic, crossing symmetric, and for pion photoproduction and Compton scattering also gauge invariant.

A convenient approach for imposing the unitarity constraint is to work with the K-matrix formalism. It results from the Bethe-Salpeter (BS) equation in the approximation where only the discontinuity part of the loop integrals is kept. In other words, the particles forming loops are taken to be on the mass shell. The S matrix in this approach is unitary and symmetric provided the K matrix is taken to be real and hermitian. Thus choosing the tree-level diagrams as the K matrix ensures (two-body) unitarity within the model space  $\pi N \oplus \gamma N$ . Two- and multi-pion production channels become important at energies  $\sqrt{s} \simeq 1.2$  GeV. These additional decay channels can be taken into account approximately by including the corresponding width in the resonance propagator.

The reaction T matrix in the channel space labeled by the index  $c=\pi N$  or  $\gamma N$  is written schematically as

$$T_{c'c} = K_{c'c} + \frac{i}{(2\pi)^4} \sum_{c''} K_{c'c''} \tilde{G}_{c''c''} T_{c''c} , \qquad (1)$$

with the appropriate two-body propagator  $\tilde{G}$ . The minimal requirement for this  $\tilde{G}$  is the correct discontinuity across the cut in the complex energy plane  $W = \sqrt{s}$ . This guarantees the unitarity of the S matrix when the kernel K is hermitian. The discontinuity can be obtained by applying the Cutkosky rules. Since the S matrix in our approach is unitary, this implies that Watson's theorem for pion photoproduction as well as the optical theorem for Compton scattering [20] are satisfied.

The K matrix is calculated from the tree-level amplitude for pion-nucleon scattering,

pion photoproduction and Compton scattering. This tree-level amplitude contains the following contributions: In the s- and u-channel exchange of the nucleon and the (four-star)  $N^*$  resonances [21]:  $P_{11}(1440)$ ,  $D_{13}(1520)$ ,  $S_{11}(1535)$ ,  $S_{11}(1650)$ ,  $P_{33}(1232)$ ,  $S_{31}(1650)$ , and  $D_{33}(1700)$ . We also include the (three-star)  $P_{11}(1710)$  listed by the PDG [21], although the PDG and the SM95 solution of the Virginia Tech  $\pi N$  PWA [22] differ significantly for the pole position, as well as the three-star  $P_{33}(1600)$  resonance, for which also the pole position is uncertain. We do not include the  $D_{13}(1700)$  listed by the PDG as a three-star resonance, but not seen in SM95. Also not taken into account are spin-5/2 resonances, in particular the  $D_{15}(1675)$  and  $F_{15}(1680)$ .

Additionally included in the tree-level amplitude are the t-channel exchange of  $\sigma(760)$  and  $\varrho(770)$  mesons for  $\pi N$  scattering,  $\pi(140)$ ,  $\varrho(770)$ , and  $\omega(781)$  mesons for pion photoproduction, and  $\pi^0(135)$  and  $\sigma(760)$  mesons for Compton scattering. The  $\sigma(760)$  could be seen as a rough manner to take into account a number of isoscalar t-channel exchanges (in particular, isoscalar two-pion,  $\varepsilon(760)$ , and "pomeron" exchange).

For the spin-3/2 resonances we take the most general coupling to the  $\pi N$  and  $\gamma N$  channels, including off-shell contributions and the full Rarita-Schwinger propagator. In this paper, we consider energies up to  $\sqrt{s} \simeq 1.7$  GeV and assume that the description in terms of nucleon resonances and t-channel meson exchanges is adequate. For higher energies, when one enters the Regge regime, one faces acute problems such as issues related to reggeizing the exchanges, duality, and possible double counting. (A recent model [23] is an example of reggeizing t-channel exchanges for pion and kaon photoproduction at energies  $E_{\gamma} > 4$  GeV.)

Above the two-pion production threshold at  $\sqrt{s} = 1.22$  GeV the nucleon resonances acquire an additional two-pion decay width. For the  $\Delta(1232)$  resonance this additional width is negligible, while for the heavier resonances we assume its energy dependence to be of the form

$$\Gamma(W) = \Gamma_0 \frac{4x}{(1+x)^2} \theta(W^2 - (m+2m_\pi)^2) , \qquad (2)$$

where

$$x = \left(\frac{W^2 - (m + 2m_{\pi})^2}{M_r^2 - (m + 2m_{\pi})^2}\right)^3 . \tag{3}$$

Here,  $M_r$  is the resonance mass. The energy dependence is chosen such as to approximately take a three-particle phase space into account. For the  $S_{11}(1535)$  resonance the additional width comes mainly from the  $N\eta$  decay channel; in this case, we take  $m_{\eta}$  instead of  $2m_{\pi}$  in Eqs. (2) and (3). The parameter  $\Gamma_0$  is chosen for each resonance equal to the decay width outside the  $\pi N$  channel. This additional width is included in the resonance propagators by changing  $M_r$  into  $M_r - i \Gamma(W)/2$ . The inclusion of this additional width gives an imaginary part to the K matrix, and accounts for all channels not included in the space  $\pi N \oplus \gamma N$ .

Concluding the description of the model, we mention that the above amplitude (for pion photoproduction and Compton scattering) obeys gauge invariance provided the K matrix is constructed to be gauge invariant. Where there are no form factors in the Born diagrams gauge invariance is satisfied, while the resonance contributions are gauge invariant by themselves also when form factors are included. However, when form factors are included in the  $\pi NN$  and/or  $\gamma NN$  vertices in the Born diagrams additional terms are needed in the

pion-photoproduction and Compton amplitudes to fulfill gauge invariance. To construct these terms we have used minimal substitution, following the method of Ref. [24]. In particular, the additional terms for pion photoproduction reproduce the Kroll-Ruderman term when the form factors are put equal to unity. We have checked explicitly that the resulting amplitudes are indeed gauge invariant numerically.

The parameters in the model have been fitted to the Virginia Tech single-energy partial-wave amplitudes [25] of Arndt and collaborators. The pion-nucleon S-, P-, and D-wave phase shifts and inelasticities for isospin I=1/2 and I=3/2 are fitted up to  $T_{\pi}=820$  MeV pion lab energy. The corresponding pion-photoproduction multipoles are fitted up to  $E_{\gamma}=805$  MeV photon lab energy. Finally, Compton scattering cross sections are also included in the fit, in particular, recent cross sections from Saskatoon and Mainz [26–30]. The limited number of free parameters (see below and Tables I and II) are strongly constrained by such a combined fit. Anecdotal results of the fit are shown in Figs. 1–3. In the following, we will focus attention to some particular details of our results, discussing in turn pion-nucleon scattering, pion photoproduction, and Compton scattering.

The pion-nucleon phase shifts are well described, although deviations start to show up at the upper limit of the energy range considered. The inclusion of an additional width in the propagator accounts rather well for the observed inelasticities for pion-nucleon scattering. However, the peculiar "saturation" of the inelasticity in some partial waves, in particular  $P_{11}$ , could not be reproduced completely. The  $P_{11}$  partial wave has more peculiar features. An attempt to describe it in our approach with one "Roper" resonance situated at approximately 1.44 GeV failed. We find that the background contribution to this  $P_{11}$  phase shift due to t- and u-channel processes is rather large, about 60° at 1.5 GeV. When a resonance at 1.44 GeV is added to such a background, one obtains a phase shift that increases rapidly to a value larger than 180°. An acceptable description of the  $P_{11}$  phase shift could be obtained by placing an additional resonance at a higher energy of  $\sqrt{s} = 1.71$  GeV.

As mentioned, off-shell form factors for the nucleon resonances have been introduced in the model. We have not investigated the importance of the detailed dependence on the invariant mass and merely adjusted the cutoff. The form factor is normalized to unity at the resonance and suppresses the resonance contribution at higher and lower invariant masses. This choice thus leads to a suppression of in particular the u-channel exchange diagrams. This could be justified in the cloudy-bag model [31] where an  $N^*$  resonance is considered as a mixture of the three-quark and  $\pi N$  scattering states (see also the discussion in Ref. [8]). The cutoff form factors are taken to depend on the invariant mass of the intermediate N or  $N^*$ , that is, on the Mandelstam variable s for s-channel exchanges and on u for u-channel exchanges, viz.

$$F(x) = \frac{\Lambda^4}{\Lambda^4 + (x - M_r^2)^2}$$
,

where  $M_r$  is the nucleon or resonance mass, and x equals s or u. In this way the crossing-symmetry properties of the amplitudes remain the same as for the case without form factors. The cutoff mass of the form factors has been set to 1.2 GeV. The results are not very sensitive to the precise value.

The  $\pi N^*N$  coupling for spin-3/2 resonances contains two parameters, the strength g and the off-shell parameter  $z_{\pi}$ , which determines the coupling to the spin-1/2 sector of

the Rarita-Schwinger propagator [13,32]. In the following, we will also use the parameter  $a_{\pi} = -(0.5+z_{\pi})$ . It is known that this off-shell parameter is important for a good description of pion photoproduction [6,10] and Compton scattering [12]. We found that the main effect of adjusting  $z_{\pi}$  is an overall shift in the spin-1/2 channel with the same isospin (and opposite parity) as the resonance channel. The influence of the off-shell parameter for the  $P_{33}(1232)$  resonance on the  $S_{31}$  partial wave and of the off-shell parameter for the  $D_{13}(1520)$  on the  $S_{11}$  and  $P_{11}$  partial waves are especially pronounced. The value of the off-shell parameter for the  $\Delta(1232)$  favored by the fit,  $a_{\pi} = -0.265$ , is close to the "Peccei value" [13,32]  $z_{\pi} = a_{\pi} = -0.25$ .

In this connection, we mention the tree-level calculations of pion photoproduction of Refs. [8,10]. In particular, in Ref. [8] u-channel contributions had to be suppressed in order to get a reasonable agreement with the pion-photoproduction data. However, the offshell coupling of all the spin-3/2 resonances were set to  $z_{\pi} = -0.25$  from the beginning. In Ref. [10], on the other hand, these parameters were determined from the data; the resonance parameters extracted were quite different from those of Ref. [8]. The authors of [10] also used a cutoff only for the u-channel diagrams, different from our prescription.

For the  $\pi N^*N$  couplings, we assume a derivative coupling of the pion. For the  $\pi NN$  case, e.g., we take the standard pseudovector Lagrangian. For the  $D_{13}(1520)$  resonance a very specific coupling was required. The general Lagrangian is, with  $R = D_{13}$ ,

$$\mathcal{L}_{\pi NR} = \frac{f_{\pi NR}}{m_{\pi}} \overline{N} \left[ (1 - \chi) \frac{i \partial \!\!\!/}{M + M_R} + \chi \right] \left( g_{\alpha\beta} + a_{\pi} \gamma_{\alpha} \gamma_{\beta} \right) \partial^{\alpha} \vec{\pi} \cdot \vec{\tau} \gamma_5 R^{\beta} + \text{ h.c.}$$
 (4)

For  $\chi = 0$ , a strong coupling to the "negative-energy" components of the nucleon results, leading to an undesirable resonance-like peak in the  $P_{13}$  partial wave. For nonzero and increasing  $\chi$  this coupling becomes more-and-more suppressed, and a reasonable description of the  $P_{13}$  phase shift is possible for  $\chi = 0.5$ .

In Table I, we list the results for the different resonances, their pole positions, one-pion couplings, and additional (two- and multi-pion) decay widths. For the spin-3/2 resonances we also give the off-shell parameters  $z_{\pi}$  and  $z_{1,\gamma}, z_{2,\gamma}$ . The one-pion couplings are in general in good agreement with those listed in the Tables of the Particle Data Group, although for the  $S_{11}(1650)$  and  $S_{31}(1620)$  they are on the high side.

The pion-nucleon coupling constant is fixed [33] to  $g_{\pi NN}=13.02$ . For the coupling constants of the  $\varrho(770)$  we get the following results:  $g_{\pi\pi\varrho}=6.07$  is taken from the two-pion decay width. In the fit we get then for the couplings to the nucleon:  $g_{\varrho NN}=4.12$  and  $\kappa_{\varrho}=1.79$ . The fit is only sensitive to the product  $g_{\pi\pi\varrho}g_{\varrho NN}$ . When we assume, following Sakurai, universal coupling of the vector mesons to the isospin current, we have  $g_{\pi\pi\varrho}=g_{\varrho NN}=5.00$ . Our result for  $\kappa_{\varrho}$  is significantly lower than the prediction  $\kappa_{\varrho}=3.71$  from vector dominance of the nucleon electromagnetic form factors. The  $\omega(781)$  coupling constant  $g_{NN\omega}$  follows from assuming SU(3) symmetry, ideal mixing of the vector mesons, and Sakurai's universality as  $g_{NN\omega}=3/2\,g_{NN\varrho}=7.50$ . Vector dominance gives  $\kappa_{\omega}=-0.12$ . The  $\sigma(760)$  couplings to the nucleon and pion are defined as in Ref. [3], viz.

$$\mathcal{L}_{NN\sigma} = -g_{NN\sigma} \, \overline{N} N \sigma \, , \quad \mathcal{L}_{\pi\pi\sigma} = -g_{\pi\pi\sigma} \, m_{\pi} \, \vec{\pi} \cdot \vec{\pi} \, \sigma \, , \tag{5}$$

where only the product  $g_{\pi\pi\sigma}g_{NN\sigma}$  appears in the  $\pi N$  scattering amplitude. We get  $g_{\pi\pi\sigma}g_{NN\sigma}=4.7$ .

In general, a good fit of the electromagnetic multipoles in pion photoproduction has been obtained. The resulting photon couplings to the different resonances can be found in Table II. The meson-photon coupling constants are determined from the decay widths [8]:  $g_{\rho^{\pm}\pi\gamma} = 0.103$ ,  $g_{\rho^{0}\pi\gamma} = 0.131$ , and  $g_{\omega\pi\gamma} = 0.313$ .

It is found from fitting the Compton scattering cross sections that the parameters of the  $\Delta N\gamma$  vertex are determined rather accurately by these data. In particular, the near-vanishing of the cross section at forward angles close to the pion-production threshold is due to a cancellation between nucleon and  $\Delta(1232)$  contributions. At large angles the  $\sigma(760)$  meson gives an important contribution to the cross section, while the t-channel contributions vanish at forward angles where t=0. The  $\pi^0$ -exchange contribution is fixed without free parameters from the axial-anomaly Lagrangian [34]

$$\mathcal{L}_{\pi^0 \gamma \gamma} = \frac{e^2 g_{\pi^0 \gamma \gamma}}{2 m_{\pi^0}} \, \varepsilon_{\mu\nu\alpha\beta} \, \partial^{\mu} A^{\nu} \, \partial^{\alpha} A^{\beta} \pi^0 \,\,, \tag{6}$$

where  $g_{\pi^0\gamma\gamma} = N_c m_{\pi^0}/12\pi^2 f_{\pi} = 0.038$  with  $N_c = 3$  and  $f_{\pi} = 92.4$  MeV. The photon coupling to the  $\sigma$ -meson is

$$\mathcal{L}_{\sigma\gamma\gamma} = \frac{e^2 g_{\sigma\gamma\gamma}}{2m_{\sigma}} \,\partial^{\mu} A^{\nu} (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}) \,\sigma \,\,, \tag{7}$$

where  $g_{\sigma\gamma\gamma}$  is determined accurately from the Compton data; we found  $g_{\sigma\gamma\gamma}g_{NN\sigma} = -5.65$ , which for  $g_{\sigma\gamma\gamma} = -0.565$  corresponds to the  $\sigma \to \gamma\gamma$  decay width  $\Gamma = 10.15$  keV.

Apart from the  $P_{33}(1232)$  and  $D_{13}(1520)$ , the  $N^*$  resonances make a negligible contribution to Compton scattering. The Compton data are sensitive mainly to the  $\Delta N\gamma$  couplings and the  $\sigma\gamma\gamma$  coupling. Since the  $\Delta N\gamma$  parameters are also strongly constrained by the pion photoproduction multipoles, a simultaneous fit of both processes is not trivial: one essentially has only the  $\sigma\gamma\gamma$  coupling to adjust in fitting Compton cross sections.

The model predicts the electic and magnetic polarizability of the proton. At the real photon point, we obtain<sup>1</sup>:  $\alpha(0) + \beta(0) = 8.23$  and  $\alpha(0) - \beta(0) = 3.05$ . Hence, our result for the electric polarizability is  $\alpha(0) = 5.64$ , which is too small. The magnetic polarizability is  $\beta(0) = 2.59$ , which agrees well with experiment [35]: the "global average" data for the proton are  $\alpha(0) = 12.1(0.8)(0.5)$  and  $\beta(0) = 2.1(0.8)(0.5)$ . It is not obvious why the electric polarizability is underestimated. This remains to be investigated, but it is likely related to the fact that the modelling of two-pion exchange by a sharp  $\sigma$ -meson is too crude. Also the fit to the Compton-scattering data leaves room for improvement. In this connection we mention calculations in Heavy Baryon ChPT where the polarizabilities come from pion-nucleon loops. The predicted values are:  $\alpha(0) = 10.5(2.0)$ ,  $\beta(0) = 3.5(3.6)$  [36,37].

In summary, we have presented results for pion-nucleon scattering, pion photoproduction, and Compton scattering using the K-matrix formalism. The model used is fully relativistic, crossing symmetric, and gauge invariant. Due to the simultaneous description of the three different processes, our model correlates a huge amount of data, resulting in a strongly constrained fit. The four-star resonances below  $\sqrt{s} \simeq 1.7$  GeV have been included explicitly and the  $\pi N^*N$  and  $\gamma N^*N$  couplings have been determined from a fit to the available partial-wave amplitudes and cross sections.

 $<sup>^{1}</sup>$ The polarizabilities are given in units  $10^{-4}$  fm $^{3}$ .

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**TABLES** 

	$M_r$		$\Gamma_0$		$g_{\pi N^*N}$		
$N^*$ resonance	Model	PDG	Model	PDG	Model	PDG	$a_{\pi}$
$P_{11}(1440)$	1520	1440	372	230	6.272	5.142	
$D_{13}(1520)$	1505	1520	90	54	1.909	1.537	-0.910
$S_{11}(1535)$	1535	1535	80	83	2.200	2.326	
$S_{11}(1650)$	1700	1650	95	45	3.630	2.375	
$P_{11}(1710)$	1710	1710	150	70	0.890	0.890	
$P_{33}(1232)$	1224	1232	_	-	2.050	2.116	-0.265
$P_{33}(1600)$	1650	1600	200	290	0.301	0.501	-0.250
$S_{31}(1620)$	1620	1620	112	112	3.641	2.319	
$D_{33}(1700)$	1650	1700	200	255	1.189	1.284	+0.280

TABLE I. Results for the different  $N^*$  resonances: pole masses, one-pion couplings, two- (and multi-)pion widths, and off-shell parameters.

	$g_{1,\gamma N^*N}$			$g_{2,\gamma N^*N}$		
$N^*$ resonance	Model	Ref. [10]	Model	Ref. [10]	$a_{1,\gamma}$	$a_{2,\gamma}$
$P_{11}(1440)$	-0.97	-0.400				
$D_{13}(1520)$	5.12	3.449	4.78	3.003	-0.06	0.95
$S_{11}(1535)$	-0.56	-0.623				
$S_{11}(1650)$	0.59	-0.205				
$P_{33}(1232)$	5.14	5.416	5.54	6.612	-0.44	2.11
$S_{31}(1620)$	-0.51	0.144				
$D_{33}(1700)$	1.74	1.895	4.75	3.921	4.19	5.76

TABLE II. Results for the different  $N^*$  resonances: photon couplings, including off-shell parameters. For the  $P_{11}(1710)$  and  $P_{33}(1600)$  no photon couplings were fitted. The comparison is to "Fit #5" (resonance pole positions) of Ref. [10].

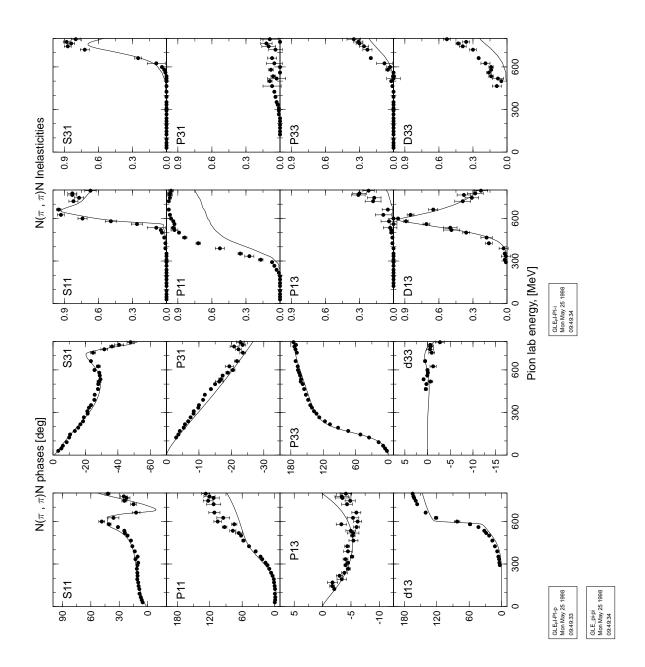


FIG. 1. Pion-nucleon S-, P-, and D-wave phase shifts and inelasticities for I=1/2 and I=3/2. The curves are our model results, the points are taken from the Virginia Tech single-energy partial-wave analysis [25].

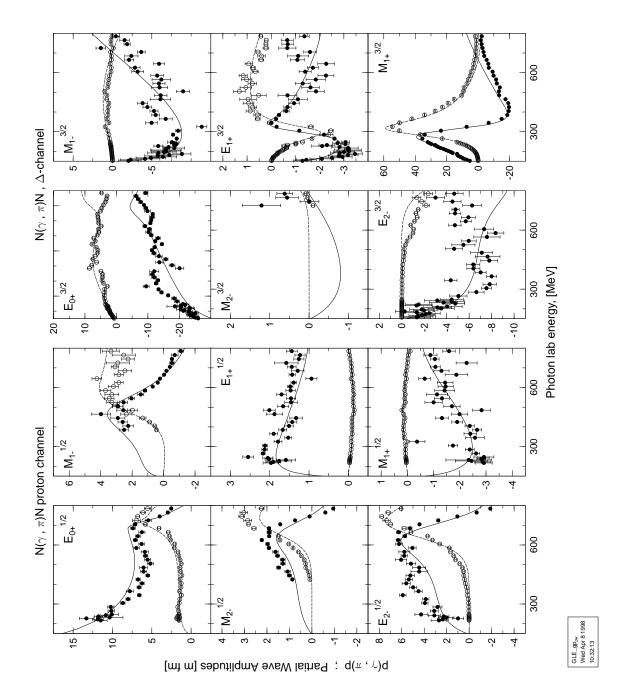


FIG. 2. Electromagnetic multipoles in mfm. The solid (dashed) curves are the real (imaginary) parts of the multipoles. The points are taken from the Virginia Tech single-energy partial-wave analysis [25].

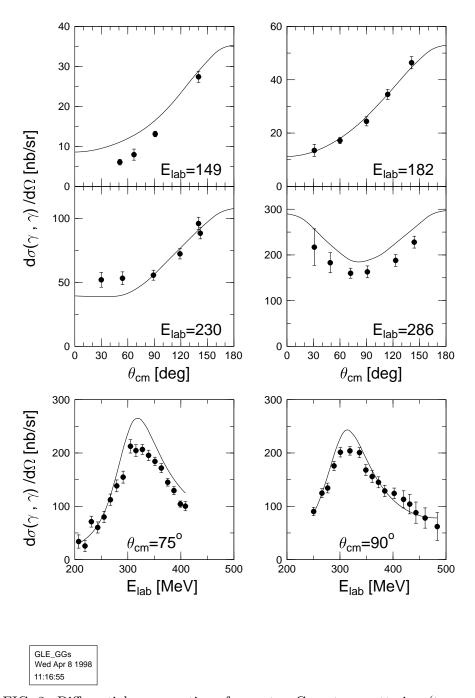


FIG. 3. Differential cross sections for proton Compton scattering (top and middle) and cross sections at  $\theta_{cm} = 75^{\circ}$  and  $\theta_{cm} = 90^{\circ}$  (bottom). The data points are from [26] (angular distributions) and from [28–30] (at fixed photon angles). The curves are the results of the present model.